

Reacting fluids in the expanding Universe: A new mechanism for entropy production

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It is shown that two reacting cosmological fluids, each of them perfect on its own, which exchange energy and momentum without preserving particle numbers, give rise to an entropy producing ‘reactive’ bulk stress of the system as a whole, as soon as the detailed balance between decay and inverse decay processes is perturbed. This demonstrates explicitly that particle generation is dynamically equivalent to an effective bulk pressure. We derive a semiquantitative formula for the corresponding new kinetic coefficient and evaluate the latter for the out-of-equilibrium decay of heavy, nonrelativistic particles into radiation. It turns out that the associated reactive bulk viscosity may be more than one order of magnitude larger than the conventional bulk viscosity, calculated, e.g., in radiative hydrodynamics.

Key words: hydrodynamics - relativity - cosmology: theory

I. INTRODUCTION

Understanding the origin of the entropy content of the Universe is one of the major problems in cosmology. Entropy production requires deviations from thermodynamical equilibrium. Such deviations typically occur if microscopic interaction time scales become comparable with cosmologically relevant time scales. Phenomenologically, entropy producing processes may be described with the help of dissipative fluid models. From the standard dissipative fluid effects, heat conductivity, shear viscosity and bulk viscosity, the latter is the most favoured one in a cosmological context since it is the only effect which is compatible with the symmetry requirements of the homogeneous and isotropic Friedmann-Lemaître-Robertson-Walker (FLRW) universes. The existence of a nonvanishing bulk viscosity for a simple relativistic gas has been known since the work of Israel [1]. A radiative bulk viscosity coefficient was derived by Weinberg [2] who also gave an analysis on its role in cosmology, followed by subsequent investigations by Straumann [3], Schweizer [4] and Udey and Israel [5]. Furthermore, bulk pressures were shown to play a role in quark-hadron transitions [6]. It was explicitly demonstrated recently [7] that the occurrence of a cosmological bulk viscosity is a general feature of any system of interacting fluids with different cooling rates in the expanding universe. The considerations of all the mentioned papers were based on the assumption that the particle numbers of each of the components are separately conserved during the interaction process. But particle number conservation is only a very special case, especially at high energies. In the present paper we consider a two-fluid universe with mutually interacting and reacting components that are allowed to exchange energy and momentum and to convert into each other. The corresponding change in the number of fluid particles is modelled by simple rate equations. In general, these reactions do not preserve the total number of fluid particles. Each of the interacting and reacting fluids is assumed to be perfect on its own. As long as the components are in thermal equilibrium and the conversion rates obey detailed balance relations, the entropy production of the system as a whole vanishes. Equilibrium situations like these are assumed to be realized at very high temperatures when the equations of state of both of the components and of the system as a whole are those for relativistic matter. As the Universe cools down, one of the subsystems, say fluid 1, becomes nonrelativistic while the other fluid, say fluid 2, continues to behave like radiation. The equations of state of the fluids are now different. Moreover, the detailed balance relations are no longer satisfied in general. The reactions may proceed preferentially in one direction, the inverse processes being suppressed. Typical cases for this kind of situation are the out-of-equilibrium decay of heavy bosons into quarks and leptons during the process of baryogenesis, or the scalar field decay into relativistic particles in the reheating phase of ‘standard’ inflationary scenarios [8]. Also, the annihilation of electrons and positrons into photons at the nucleosynthesis energy scale belongs to this category.

The purpose of this paper is to demonstrate that *the difference in the equations of state of the fluid components and the simultaneous deviation from the detailed balance in the rate equations for the interfluid reactions give rise to a new entropy producing cosmological effect that manifests itself as a ‘reactive’ bulk pressure.* We shall derive

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here a semiquantitative formula for this effect for small deviations from equilibrium and apply it to a typical out-of-equilibrium situation in the early universe.

To point out the difference to conventional bulk pressures, calculated, e.g., in radiative hydrodynamics, we emphasize that the physical reason for the effect under consideration here is twofold. The first reason is that because of their different equations of state the fluid subsystems are subject to different cooling rates in the expanding Universe. The second reason are the deviations from the equilibrium balances between the decay and inverse decay reactions of the fluids. It is the combination of both these causes that produces the effect to be discussed in the present paper. For conventional bulk viscosities, differences in the cooling rates alone are sufficient [5,7].

The described effect implies that in a reacting mixture the generation of particles is dynamically equivalent to an effective viscous pressure. For a decay rate comparable with the expansion rate we show this effect to be more than one order of magnitude larger than the corresponding effect due to a conventional bulk viscosity.

The structure of the paper is as follows: Section 2 establishes the basic theory of two interacting and reacting fluids with generally different temperatures. Assuming that the interactions establish an approximate equilibrium between the fluids, we investigate the same system within an effective one-temperature model in section 3. Comparing both descriptions, specifically the respective expressions for the entropy production, we derive a general, semiquantitative formula for the reactive bulk pressure of the system as a whole in section 4. Section 5 presents an application of this formula to a mixture of a nonrelativistic and a relativistic fluid and compares the reactive bulk viscosity coefficient with the conventional one. A brief summary of the paper and a few comments on cosmological applications are given in section 6.

Units have been chosen so that $c = k_B = \hbar = 1$.

II. THE TWO-FLUID MODEL

The energy-momentum tensor T^{ik} of the cosmic substratum is assumed to split into two perfect fluid parts,

$$T^{ik} = T_1^{ik} + T_2^{ik} , \quad (1)$$

with $(A = 1, 2)$

$$T_A^{ik} = \rho_A u^i u^k + p_A h^{ik} . \quad (2)$$

ρ_A is the energy density and p_A is the equilibrium pressure of species A . For simplicity we assume that both components share the same 4-velocity u^i . The quantity h^{ik} is the projection tensor $h^{ik} = g^{ik} + u^i u^k$. The particle flow vector N_A^i of species A is defined as

$$N_A^i = n_A u^i , \quad (3)$$

where n_A is the particle number density. We are interested in situations where neither the particle numbers nor the energy-momenta of the components are separately conserved, i.e., conversion of particles and exchange of energy and momentum between the components are admitted. The balance laws for the particle numbers are

$$N_{A;i}^i = \dot{n}_A + \Theta n_A = n_A \Gamma_A , \quad (4)$$

where $\Theta \equiv u_{;i}^i$ is the fluid expansion and Γ_A is the rate of change of the number of particles of species A . There is particle production for $\Gamma_A > 0$ and particle decay for $\Gamma_A < 0$, respectively. For $\Gamma_A = 0$, we have separate particle number conservation.

Interactions between the fluid components amount to the mutual exchange of energy and momentum. Consequently, there will be no local energy-momentum conservation for the subsystems separately. Only the energy-momentum tensor of the system as a whole is conserved.

Denoting the loss- and source-terms in the separate balances by t_A^i , we write

$$T_{A;k}^{ik} = -t_A^i , \quad (5)$$

implying

$$\dot{\rho}_A + \Theta(\rho_A + p_A) = u_a t_A^a , \quad (6)$$

and

$$(\rho_A + p_A) \dot{u}^a + p_{A,k} h^{ak} = -h_i^a t_A^i . \quad (7)$$

All the considerations to follow will be independent of the specific structure of the t_A^i . In other words, there are no limitations on the strength or the structure of the interaction.

Each component is governed by a separate Gibbs equation

$$T_A ds_A = d\frac{\rho_A}{n_A} + p_A d\frac{1}{n_A} , \quad (8)$$

where s_A is the entropy per particle of species A . Using eqs.(4) and (6) one finds for the time behaviour of the entropy per particle

$$n_A T_A \dot{s}_A = u_a t_A^a - (\rho_A + p_A) \Gamma_A . \quad (9)$$

With nonvanishing source terms in the balances for n_A and ρ_A , the change in the entropy per particle is different from zero in general. Below we shall deal with the special case that the terms on the right-hand side of eq.(9) cancel.

The equations of state of the fluid components are assumed to have the general form

$$p_A = p_A(n_A, T_A) \quad (10)$$

and

$$\rho_A = \rho_A(n_A, T_A) , \quad (11)$$

i.e., particle number densities n_A and temperatures T_A are regarded as the basic thermodynamical variables. The temperatures of the fluids are different in general.

Differentiating relation (11), using the balances (4) and (6) as well as the general relation

$$\frac{\partial \rho_A}{\partial n_A} = \frac{\rho_A + p_A}{n_A} - \frac{T_A}{n_A} \frac{\partial p_A}{\partial T_A} , \quad (12)$$

that follows from the requirement that the entropy is a state function, we find the following expression for the temperature behaviour [9–11]:

$$\dot{T}_A = -T_A (\Theta - \Gamma_A) \frac{\partial p_A / \partial T_A}{\partial \rho_A / \partial T_A} + \frac{u_a t_A^a - \Gamma_A (\rho_A + p_A)}{\partial \rho_A / \partial T_A} . \quad (13)$$

Both the source terms Γ_A and $u_a t_A^a$ in the balances (4) and (6) backreact on the temperature. The numerator of the second term on the right-hand side of eq.(13) coincides with the right-hand side of eq.(9), i.e., the corresponding terms disappear in the special case $\dot{s}_A = 0$.

For $\Gamma_A = u_a t_A^a = 0$ and with $\Theta = 3\dot{R}/R$, where R is the scale factor of the Robertson-Walker metric, the equations of state $p_r = n_r k T_r$, $\rho_r = 3n_r k T_r$ for radiation (subscript ‘r’) reproduce the well known $T_r \sim R^{-1}$ behaviour. With $p_m = n_m k T_m$, $\rho_m = n_m m + \frac{3}{2} n_m k T_m$ one recovers $T_m \sim R^{-2}$ for matter (subscript ‘m’).

The entropy flow vector S_A^a is defined by

$$S_A^a = n_A s_A u^a , \quad (14)$$

and the contribution of component A to the entropy production density becomes

$$\begin{aligned} S_{A;a}^a &= n_A s_A \Gamma_A + n_A \dot{s}_A \\ &= \left(s_A - \frac{\rho_A + p_A}{n_A T_A} \right) n_A \Gamma_A + \frac{u_a t_A^a}{T_A} , \end{aligned} \quad (15)$$

where relation (9) has been used.

According to eq.(5) the condition of energy-momentum conservation for the system as a whole,

$$(T_1^{ik} + T_2^{ik})_{;k} = 0 , \quad (16)$$

implies

$$t_1^a = -t_2^a . \quad (17)$$

There is no corresponding condition, however, for the particle number balance as a whole. Defining the integral particle number density n as

$$n = n_1 + n_2 , \quad (18)$$

we have

$$\dot{n} + \Theta n = n\Gamma , \quad (19)$$

where

$$n\Gamma = n_1\Gamma_1 + n_2\Gamma_2 . \quad (20)$$

Γ is the rate by which the total particle number n changes. We do *not* require Γ to be zero since total particle number conservation is only a very special case, especially at high energies.

The entropy per particle is [12]

$$s_A = \frac{\rho_A + p_A}{n_A T_A} - \frac{\mu_A}{T_A} , \quad (21)$$

where μ_A is the chemical potential of species A . Introducing the last expression into eq.(15) yields

$$S_{A;a}^a = -\frac{\mu_A}{T_A} n_A \Gamma_A + \frac{u_a t_A^a}{T_A} . \quad (22)$$

For the total entropy production density

$$S_{;a}^a = S_{1;a}^a + S_{2;a}^a \quad (23)$$

we obtain

$$S_{;a}^a = -\frac{\mu_2}{T_2} n\Gamma - \left(\frac{\mu_1}{T_1} - \frac{\mu_2}{T_2} \right) n_1 \Gamma_1 + \left(\frac{1}{T_1} - \frac{1}{T_2} \right) u_a t_1^a . \quad (24)$$

The condition $S_{;a}^a = 0$ requires the well-known equilibrium conditions (see, e.g., [13] (chapter 5))

$$\mu_1 = \mu_2 , T_1 = T_2 , \quad (25)$$

as well as $\Gamma = 0$.

From now on we assume that the source terms on the right-hand side of eq.(9) cancel among themselves, i.e., that the entropy per particle of each of the components is preserved. The particles decay or come into being with a fixed entropy s_A . This ‘isentropy’ condition amounts to the assumption that the particles at any stage are amenable to a perfect fluid description. With $\dot{s}_A = 0$, via eq.(9) equivalent to

$$u_a t_A^a = (\rho_A + p_A) \Gamma_A , \quad (26)$$

the expression (13) for the temperature behaviour and the expression (24) for the entropy production density simplify considerably. The condition (26) establishes a relation between the source terms in the balances (4) and (6) which originally are independent quantities.

Combining eqs.(17) and (26), one has

$$u_a t_1^a = (\rho_1 + p_1) \Gamma_1 = -u_a t_2^a = -(\rho_2 + p_2) \Gamma_2 , \quad (27)$$

which provides us with a relation between the rates Γ_1 and Γ_2 :

$$\Gamma_2 = -\frac{\rho_1 + p_1}{\rho_2 + p_2} \Gamma_1 . \quad (28)$$

Inserting the last relation into equation (20) yields

$$n\Gamma = n_1 \Gamma_1 h_1 \left[\frac{1}{h_1} - \frac{1}{h_2} \right] . \quad (29)$$

The quantities $h_A \equiv (\rho_A + p_A)/n_A$ are the enthalpies per particle. Total particle number conservation, i.e., $\Gamma = 0$ is only possible if the enthalpies per particle of both components coincide, i.e., for $h_1 = h_2$.

With the relations (26) and (29) the entropy production density (24) becomes

$$S_{;a}^a = (\rho_1 + p_1) \left[\frac{n_1 s_1}{\rho_1 + p_1} - \frac{n_2 s_2}{\rho_2 + p_2} \right] \Gamma_1 = n_1 \Gamma_1 h_1 \left[\frac{s_1}{h_1} - \frac{s_2}{h_2} \right]. \quad (30)$$

We emphasize that according to the equations of state (10) and (11) the quantities ρ_1, p_1 and s_1 depend on T_1 , while ρ_2, p_2 and s_2 depend on T_2 . In general, we have $T_1 \neq T_2$.

From the expression (30) it is obvious that a decay of fluid 1 into fluid 2 particles is accompanied by a nonvanishing entropy production unless $s_1/s_2 = h_1/h_2$. For $\mu_1 = \mu_2 = 0$ the latter condition reduces to $T_1 = T_2$. This situation corresponds to the case that all the entropy of fluid 1 is transferred into that of fluid 2. On this basis one usually discusses, e.g., the cosmological electron-positron annihilation at an energy scale of about 1 MeV ([8,13–15].)

III. THE EFFECTIVE ONE-TEMPERATURE MODEL

There exists a more familiar, alternative description of a two-component fluid close to equilibrium, that is based on a single Gibbs-equation for the system as a whole:

$$T ds = d\frac{\rho}{n} + p d\frac{1}{n} - (\mu_1 - \mu_2) d\frac{n_1}{n}, \quad (31)$$

where s is the entropy per particle. The temperature T is the equilibrium temperature of the whole system. The temperatures T_1 and T_2 of the previous section do not appear as variables in the present effective one-temperature description. An (approximate) equilibrium for the entire system is assumed to be established through the interactions between the subsystems on the right-hand side of the balances (6) and (7). Furthermore, we assume that analogous to the equations of state (10) and (11) the cosmic fluid as a whole is characterized by equations of state

$$p = p(n, n_1, T) \quad (32)$$

and

$$\rho = \rho(n, n_1, T), \quad (33)$$

where p is the equilibrium pressure and ρ is the energy density of the system as a whole. From the special case of a mixture of radiation and nonrelativistic matter discussed below eq.(13) it is obvious that the overall quantities p and ρ depend on two independent number densities which we have chosen to be n and n_1 . As long as the pressures are those for classical gases, i.e., $p_A = n_A T$, the equilibrium pressure p of the system as a whole depends on $n = n_1 + n_2$ only and the separate dependence on n_1 on the right-hand side of eq.(32) may be omitted.

If in the expressions (21) for the entropies per particle the temperatures T_1 and T_2 are identified among themselves and with T , and $\rho(T) = \rho_1(T) + \rho_2(T)$ as well as $p(T) = p_1(T) + p_2(T)$ are used, the description based on relation (31) is consistent with the description relying on the relations (8) for $ns(T) = n_1 s_1(T) + n_2 s_2(T)$.

The equilibrium temperature T is *defined* by [5,7]

$$\rho_1(n_1, T_1) + \rho_2(n_2, T_2) = \rho(n, n_1, T). \quad (34)$$

As it was shown in [7] for $\Gamma = \Gamma_1 = \Gamma_2 = 0$, this generally implies

$$p_1(n_1, T_1) + p_2(n_2, T_2) \neq p(n, n_1, T). \quad (35)$$

For perfect fluids 1 and 2 with separately conserved particle numbers the difference between both sides of the latter inequality is a viscous pressure. Its existence is a consequence of the different temperature evolution laws of the subsystems. As it is obvious from eq.(13) the cooling rate \dot{T}_1/T_1 is different from \dot{T}_2/T_2 even for $\Gamma_1 = \Gamma_2 = 0$ if the subsystems are governed by different equations of state. The expansion of the Universe tends to increase the difference between T_1 and T_2 , i.e., to drive the system as a whole away from equilibrium. This manifests itself as a bulk viscous pressure [7].

We shall show here that nonvanishing source terms Γ_A in the particle number balances (4) give rise to a new type of effective bulk pressure, i.e., to enlarged entropy production.

In order to find an explicit description of this phenomenon we shall use the following line of arguing. We *anticipate* that deviations from detailed balance, i.e., $\Gamma_A \neq 0$, leading to $\Gamma \neq 0$ in general, will generate an effective ‘reactive’ bulk pressure π_{react} . For the corresponding energy-momentum tensor of the system as a whole we write from the outset

$$T^{ik} = \rho u^i u^k + (p + \pi_{react}) h^{ik} . \quad (36)$$

To separate the reactive bulk pressure from any other dissipative phenomenon, we have ignored here the possibility of conventional bulk pressures, dealt with elsewhere [7] and of nonvanishing heat fluxes and shear stresses in inhomogeneous and anisotropic cosmological models. The relations $T^{ik}_{;k} = 0$ then imply the energy balance

$$\dot{\rho} + \Theta(\rho + p + \pi_{react}) = 0 \quad (37)$$

for the effective one-temperature description. *Afterwards* we shall determine π_{react} by the consistency requirement of the one-temperature description on the basis of eqs.(31), (4), (19), and (37), with the two-temperature description based on eqs.(8), (4), (6), (17), and (26). More specifically, π_{react} will be obtained by comparing eq.(23), leading to the two-temperature expression (30) for the entropy production density with the corresponding expression for $S^a_{;a}$ to be calculated below within the one-temperature picture.

From the Gibbs-equation (31) one finds for the change in the entropy per particle

$$n\dot{s} = -\frac{\Theta}{T}\pi_{react} - \frac{\rho + p}{T}\Gamma - \frac{n_1 n_2}{n} \left(\frac{\mu_1 - \mu_2}{T} \right) [\Gamma_1 - \Gamma_2] . \quad (38)$$

Even for $\dot{s}_1 = \dot{s}_2 = 0$ we have $\dot{s} \neq 0$ in general. From $S^a = nsu^a$ the expression for the entropy production density is

$$S^a_{;a} = ns\Gamma + n\dot{s} . \quad (39)$$

Introducing here eq.(38), we find

$$S^a_{;a} = -\frac{\Theta}{T}\pi_{react} - \frac{n\mu}{T}\Gamma - \frac{n_1 n_2}{n} \left(\frac{\mu_1 - \mu_2}{T} \right) [\Gamma_1 - \Gamma_2] , \quad (40)$$

where

$$\mu = \frac{\rho + p}{n} - Ts \quad (41)$$

is the effective one-component chemical potential.

Applying in eq.(40) the relation (28) as well as the decomposition

$$n\mu = n_1\mu_1 + n_2\mu_2 \quad (42)$$

for the chemical potential, the entropy production density may be written as

$$S^a_{;a} = -\frac{\rho_1 + p_1}{T}\Gamma_1 \left(\frac{n_1\mu_1}{\rho_1 + p_1} - \frac{n_2\mu_2}{\rho_2 + p_2} \right) - \frac{\Theta}{T}\pi_{react} . \quad (43)$$

Here we used $\rho(T) = \rho_1(T) + \rho_2(T)$ and $p(T) = p_1(T) + p_2(T)$, keeping in mind that all the quantities, including those of the subsystems, depend on T , *not* on T_A . There is only one temperature in this kind of description, namely T .

The expression (43) for the entropy production density is a sum of terms each being a product of thermodynamical ‘fluxes’ and thermodynamical ‘forces’. According to the standard phenomenological theory, the requirement $S^a_{;a} \geq 0$ may be fulfilled by linear relations between ‘fluxes’ and ‘forces’. The new aspect here, compared with the standard one is, that one has two scalar fluxes, namely π_{react} and Γ_1 . This will generally result in cross effects between both phenomena. A related discussion of chemical reactions in the expanding Universe was given in [16].

Within the causal thermodynamics of irreversible processes, both fluxes become dynamical degrees of freedom on their own and one arrives at coupled evolution equations for π_{react} and Γ_1 . Investigations along these lines have been carried out for a one-component system by Gariel and Le Denmat [17].

In the present paper we do not follow this kind of arguing of the phenomenological theory. Instead, we shall find an explicit expression for π_{react} in terms of Γ_A with the help of semiquantitative arguments. The quantity connecting π_{react} and Γ_1 represents a new kinetic coefficient, corresponding to the mentioned cross effect.

Strictly speaking, the computation of π_{react} has to come from kinetic theory, which, however, is beyond the scope of the present paper. Our derivation may be regarded as intermediate between the phenomenological and kinetic levels of description.

From eq.(23) the alternative expression for the entropy production density $S_{;a}^a$ within the two-temperature model becomes

$$S_{;a}^a = n_1 s_1 \Gamma_1 + n_2 s_2 \Gamma_2 , \quad (44)$$

with $s_1 = s_2(T_1)$ and $s_2 = s_2(T_2)$ from (21), where we assumed each of the subfluids to be perfect on their own, i.e., $\dot{s}_1 = \dot{s}_2 = 0$. The two-temperature formula (44) has to be consistent with the one-temperature formula (43). This consistency requirement will be used to determine π_{react} . The obvious difficulty one encounters here is that one has to deal with three generally different temperatures: The temperatures T_1 and T_2 of the components 1 and 2 in formula (44) and the equilibrium temperature T of the system as a whole in (43). The dynamics of T_1 and T_2 are given by eqs.(13) with eqs.(26). In order to compare the expressions (43) and (44) for the entropy production we have to find a corresponding law for T . Such a law may be obtained via similar steps that led us to the expression (13). There exists, however, the following complication. Because of the additional dependence of ρ on n_1 one has now three partial derivatives of ρ : $(\partial\rho/\partial T)_{n,n_1}$, $(\partial\rho/\partial n)_{T,n_1}$, and $(\partial\rho/\partial n_1)_{n,T}$. The requirement that s is a state function now leads to ([18,19])

$$\frac{\partial\rho}{\partial n} = \frac{\rho+p}{n} - \frac{T}{n} \frac{\partial p}{\partial T} - \frac{n_1}{n} \left[(\mu_1 - \mu_2) - T \frac{\partial}{\partial T} (\mu_1 - \mu_2) \right] , \quad (45)$$

generalizing relation (12), and the additional relation

$$\frac{\partial\rho}{\partial n_1} = \mu_1 - \mu_2 - T \frac{\partial}{\partial T} (\mu_1 - \mu_2) . \quad (46)$$

Using the Gibbs-Duhem relations

$$dp_A = n_A s_A dT_A + n_A d\mu_A \quad (47)$$

for $T_A = T$ together with eq.(21), one finds

$$\mu_A - T \frac{\partial\mu_A}{\partial T} = \frac{\rho_A}{n_A} . \quad (48)$$

Consequently, the relations (45) and (46) may be written as

$$\frac{\partial\rho}{\partial n} = \frac{\rho+p}{n} - \frac{T}{n} \frac{\partial p}{\partial T} - \frac{n_1}{n} \left[\frac{\rho_1}{n_1} - \frac{\rho_2}{n_2} \right] \quad (49)$$

and

$$\frac{\partial\rho}{\partial n_1} = \frac{\rho_1}{n_1} - \frac{\rho_2}{n_2} , \quad (50)$$

with $\rho_1 = \rho_1(T)$ and $\rho_2 = \rho_2(T)$, since we are within the one-temperature description.

Differentiating eq.(33), using eqs.(4), (19), (37), (49), and (50), we obtain

$$\begin{aligned} \frac{\partial\rho}{\partial T} \dot{T} = & -(\Theta - \Gamma) T \frac{\partial p}{\partial T} - [\Theta \pi_{react} + \Gamma(\rho + p)] \\ & - \frac{n_1 n_2}{n} \left(\frac{\rho_1}{n_1} - \frac{\rho_2}{n_2} \right) [\Gamma_1 - \Gamma_2] . \end{aligned} \quad (51)$$

Applying here eq.(28), following from $\dot{s}_1 = \dot{s}_2 = 0$, and restricting ourselves from now on to classical fluids with $p_A = n_A T$, we find the following evolution law for the equilibrium temperature:

$$\frac{\dot{T}}{T} = -(\Theta - \Gamma) \frac{\partial p}{\partial \rho} - \frac{\Theta \pi_{react}}{T \partial \rho / \partial T} , \quad (52)$$

where the abbreviation

$$\frac{\partial p}{\partial \rho} \equiv \frac{\partial p / \partial T}{\partial \rho / \partial T} \quad (53)$$

was used.

The temperature law (52) is a new result. It describes the behaviour of the equilibrium temperature of an interacting and reacting two-fluid mixture with nonconserved total particle number. It is different in its structure from the laws (13) with (26) because of the π_{react} term. The result (52) differs also from previous one-component approaches to particle production in the expanding Universe [10,11,20]. The point here is that the ‘isentropy’ conditions $\dot{s}_A = 0$, equivalent to relation (26) of the present paper are physically different from the case of ‘adiabatic’ particle production in [10,11,20], characterized by $\dot{s} = 0$. We conclude that ‘adiabatic’ particle production in the sense of [10,11,20] is not consistent with the requirement of a perfect fluid behaviour of the subsystems in a reacting two-component mixture.

IV. THE EFFECTIVE VISCOUS PRESSURE

With the formulae (13) and (52) we know the evolution laws for all three temperatures. We are now prepared to compare the expressions (43) and (44) for the entropy production densities. To that purpose we write $s_1 = s_1(T_1)$ and $s_2 = s_2(T_2)$ as ($A = 1, 2$)

$$s_A(T_A) = s_A(T) + \Delta s_A. \quad (54)$$

Using this splitting in eq.(44) and comparing with eq.(43) yields

$$-\frac{\Theta}{T} \pi_{react} = \Gamma_1 n_1 \Delta s_1 + \Gamma_2 n_2 \Delta s_2. \quad (55)$$

Since according to eqs.(13) and (52) the temperatures T_A and T are different, the quantities $\Delta s_A = s_A(T_A) - s_A(T)$ will be nonvanishing in general. However, as long as the detailed balance relations $\Gamma_A = 0$ are fulfilled, π_{react} vanishes even for $\Delta s_A \neq 0$. (We recall that this case leads to conventional bulk stresses, discussed elsewhere [7]).

In order to find an explicit expression for π_{react} we use the following heuristic mean free time argument (cf. [7]). Let τ be the characteristic mean free time for the interaction between both components, generally symbolized by t_A^a in eq.(5). The time τ is assumed to be much larger than any characteristic interaction time within each of the fluids. Consequently, the latter may be regarded as perfect on time scales of the order of τ . The interaction between the fluids is pictured by ‘collisional’ events, where τ plays the role of a mean free time between these ‘collisions’. During the time interval τ , i.e., between subsequent interfluid interaction events, both components then evolve according to their internal perfect fluid dynamics, given by eqs.(4), (6) and (13) with condition (26).

Let us now assume that at $t = t_0$ there exist deviations from detailed balance due to the circumstance that one of the components, say fluid 1, becomes nonrelativistic and starts to decay into the relativistic fluid 2. Let us further assume that both components were in thermal equilibrium at $t = t_0$ due to their mutual interaction, i.e., $T_1(t_0) = T_2(t_0) = T(t_0)$, equivalent to $\Delta s_1(t_0) = \Delta s_2(t_0) = \pi_{react}(t_0) = 0$. Until a subsequent interaction event after a mean free time τ between the ‘collisions’ the components evolve effectively free due to their perfect fluid dynamics. According to the temperature laws (13) and (52), the temperatures T_1 , T_2 and T evolve differently for $t > t_0$. At the time t with $t - t_0 \leq \tau$ we will have $T \neq T_A$ in general with $T_1 \neq T_2$. Consequently, the quantities Δs_A will no longer vanish at the time t . Nonvanishing values of Δs_A , however, together with $\Gamma_A \neq 0$, will give rise to an effective bulk pressure according to eq.(55).

Up to first order in the temperature deviations we may write

$$\Delta s_A = (T_A - T) \left(\frac{\partial s_A}{\partial T} \right)_{n_A} + \dots \quad (56)$$

Applying the general formula

$$\frac{\partial s_A}{\partial T} = \frac{1}{n_A T_A} \frac{\partial \rho_A}{\partial T_A}, \quad (57)$$

that follows from the Gibbs-Duhem relation (47), we get

$$-\Theta \pi_{react} = n_1 \Gamma_1 c_v^{(1)} (T_1 - T) + n_2 \Gamma_2 c_v^{(2)} (T_2 - T), \quad (58)$$

where

$$c_v^{(A)} \equiv \frac{1}{n_A} \frac{\partial \rho_A}{\partial T} \quad (59)$$

are the specific heats of the components. Introducing the relation (28) between the rates Γ_1 and Γ_2 , eq.(58) may be written as

$$\begin{aligned} -\Theta \pi_{react} &= \Gamma_1 (\rho_1 + p_1) \left[\frac{n_1 c_v^{(1)} (T_1 - T)}{\rho_1 + p_1} - \frac{n_2 c_v^{(2)} (T_2 - T)}{\rho_2 + p_2} \right] \\ &= n_1 \Gamma_1 h_1 \left[\frac{c_v^{(1)} (T_1 - T)}{h_1} - \frac{c_v^{(2)} (T_2 - T)}{h_2} \right]. \end{aligned} \quad (60)$$

We recall that this reactive bulk pressure is a cross effect between the deviation from detailed balance, represented by $\Gamma_1 \neq 0$, and the deviation from thermal equilibrium, i.e., $T_1 \neq T_2 \neq T$. The conventional bulk pressure is only due to the different cooling rates. Here, both deviations from the detailed balance and from the thermal equilibrium are necessary.

Let again component 1 be the decaying component (i.e., $\Gamma_1 < 0$) with an equation of state close to that for matter. Because of $\partial p_1 / \partial \rho_1 \geq \partial p / \partial \rho \geq \partial p_2 / \partial \rho_2$, the temperature T_1 cools off faster than the equilibrium temperature T , while T_2 decreases slower than T [cf.(13), (52)]. Consequently, we expect $T_1 < T$ and $T_2 > T$. In total, the result according to eq.(60) will be a negative bulk pressure corresponding to a positive effective bulk viscosity, in accordance with the second law of thermodynamics.

Restricting ourselves to small temperature differences we have

$$T_A = T_A(t_0) + (t - t_0) \dot{T}_A + \dots, \quad (61)$$

and

$$T = T(t_0) + (t - t_0) \dot{T} + \dots. \quad (62)$$

Here, \dot{T}_A and \dot{T} have to be evaluated at $t = t_0$. While \dot{T}_A may be taken from eq.(13) immediately, we have still to show that $\dot{T}(t_0)$ is consistent with $\pi_{react}(t_0) = 0$. This may be done by applying the relation (34) that defines the equilibrium temperature T at $t > t_0$, keeping in mind that the interaction between the fluids, generally given by the source terms in the balances (6) and (7), is modelled by ‘collisional’ events, with a mean free time τ between the ‘collisions’. During the interval $(t - t_0) \leq \tau$, i.e., between subsequent interaction events, both components are effectively free, i.e., governed by their internal perfect fluid dynamics according to eqs.(4), (6), (13), and (26). Assume again that through the interaction an element of the cosmic fluid is in equilibrium at a proper time t_0 at a temperature $T(t_0) = T_1(t_0) = T_2(t_0)$ with $p(t_0) = p_1(t_0) + p_2(t_0)$. Here, $p(t_0)$ and $p_A(t_0)$ are shorts for $p[n(t_0), T(t_0)]$ and $p_A[n_A(t_0), T_A(t_0)]$, respectively. Using the condition (34) at the proper time t up to first order in $t - t_0$, i.e., with

$$\rho_A(t) = \rho_A(t_0) + (t - t_0) \dot{\rho}_A(t_0) + \dots \quad (63)$$

and

$$\rho(t) = \rho(t_0) + (t - t_0) \dot{\rho}(t_0) + \dots \quad (64)$$

where $\rho_A(t_0) \equiv \rho_A[n_A(t_0), T_A(t_0)]$ and $\rho(t_0) \equiv \rho[n(t_0), n_1(t_0), T(t_0)]$, applying eq.(6) on the left-hand side of eq.(34) and using the relations (49) and (50) on its right-hand side, one finds

$$\dot{T}(t_0) = -T(\Theta - \Gamma) \frac{\partial p}{\partial \rho}, \quad (65)$$

where all the quantities on the right-hand side of the last equation have to be taken at the point $t = t_0$. Comparing with the temperature law (52), this proves the consistency of our initial configuration with $\pi_{react}(t_0) = 0$ for $T_1(t_0) = T_2(t_0) = T(t_0)$, i.e., $\Delta s_A = 0$ [cf. eq.(55)] and $\Gamma_A(t_0) \neq 0$.

Applying now the evolution laws (13) and (65) at $t = t_0$ we get for the first-order temperature differences

$$T_A - T = -\Theta T(t - t_0) \left[\left(1 - \frac{\Gamma_A}{\Theta}\right) \frac{\partial p_A}{\partial \rho_A} - \left(1 - \frac{\Gamma}{\Theta}\right) \frac{\partial p}{\partial \rho} \right] + \dots. \quad (66)$$

The first-order approximation is valid as long as the conditions

$|\Theta - \Gamma_A|(t - t_0) \leq 1$ are fulfilled. Recalling that an effective one-fluid description requires $\tau < H^{-1}$, where $H = \Theta/3$ is the Hubble parameter, one may discuss three different cases.

(i) $|\Gamma_A|^{-1} < \tau < H^{-1}$. There is a considerable change of the particle numbers on time scales of the order of the mean free time between the interfluid interaction events.

(ii) $\tau < H^{-1} < |\Gamma_A|^{-1}$. Here, the characteristic times $|\Gamma_A|^{-1}$ for the particle number changes are larger than the Hubble time.

(iii) $\tau < |\Gamma_A|^{-1} < H^{-1}$. This case is intermediate between (i) and (ii). The particle numbers change substantially during one Hubble time but are approximately constant on scales of the order of τ .

Inserting the temperature differences (66) into eq.(60), the reactive viscous pressure may be written as

$$\begin{aligned} \pi_{react} = & \Gamma_1 \eta T \frac{\partial \rho_1}{\partial T} \left[\left(1 - \frac{\Gamma_1}{\Theta} \right) \frac{\partial p_1}{\partial \rho_1} - \left(1 - \frac{\Gamma}{\Theta} \right) \frac{\partial p}{\partial \rho} \right] \\ & + \Gamma_2 \eta T \frac{\partial \rho_2}{\partial T} \left[\left(1 - \frac{\Gamma_2}{\Theta} \right) \frac{\partial p_2}{\partial \rho_2} - \left(1 - \frac{\Gamma}{\Theta} \right) \frac{\partial p}{\partial \rho} \right], \end{aligned} \quad (67)$$

where $\eta \equiv t - t_0$. It can now be seen explicitly that π_{react} is always negative. For ‘ordinary’ matter $\partial p_A / \partial \rho_A$ lies in the range $1/3 \leq \partial p_A / \partial \rho_A \leq 2/3$. The lower limit corresponds to radiation, the upper one to matter. $\partial p / \partial \rho$ will take a value intermediate between $\partial p_1 / \partial \rho_1$ and $\partial p_2 / \partial \rho_2$. The typical case is that massive particles decay into relativistic ones. Identifying again fluid 1 with the massive decaying component, we have $\Gamma_1 < 0$. Furthermore, $\partial p / \partial \rho \leq \partial p_1 / \partial \rho_1$ and $\partial p / \partial \rho \geq \partial p_2 / \partial \rho_2$. Because of $\partial p_1 / \partial \rho_1 > \partial p / \partial \rho$ one has $(1 - \Gamma_1 / \Theta) \partial p_1 / \partial \rho_1 > (1 - \Gamma / \Theta) \partial p / \partial \rho$, since a negative Γ_1 enlarges the left-hand side of this inequality and a positive Γ diminishes its right-hand side. In the second term one has $\partial p_2 / \partial \rho_2 < \partial p / \partial \rho$. The relations (28) and (29) may be combined into

$$\Gamma = \frac{n_2}{n} \left[1 - \frac{h_2}{h_1} \right] \Gamma_2. \quad (68)$$

One has $\Gamma < \Gamma_2$ for any $\Gamma_2 > 0$. It follows that the factor $(1 - \Gamma_2 / \Theta)$ reduces the $\partial p_2 / \partial \rho_2$ term more than $(1 - \Gamma / \Theta)$ reduces the $\partial p / \partial \rho$ term. Consequently, $(1 - \Gamma_2 / \Theta) \partial p_2 / \partial \rho_2 < (1 - \Gamma / \Theta) \partial p / \partial \rho$. Both contributions to π_{react} in the expression (67) are negative, i.e., the reactive bulk viscosity coefficient is positive, as is required by the second law of thermodynamics. Inserting the expressions (28) and (59) into equation (67), the reactive viscous pressure may be written as

$$\begin{aligned} \pi_{react} = & \Gamma_1 \eta n_1 h_1 T \left[\frac{c_v^{(1)}}{h_1} \left(\frac{\partial p_1}{\partial \rho_1} - \frac{\partial p}{\partial \rho} \right) - \frac{c_v^{(2)}}{h_2} \left(\frac{\partial p_2}{\partial \rho_2} - \frac{\partial p}{\partial \rho} \right) \right] \\ & - \Gamma_1 \eta \frac{\Gamma_1}{\Theta} n_1^2 h_1^2 T \left[\frac{c_v^{(1)}}{n_1 h_1^2} \left(\frac{\partial p_1}{\partial \rho_1} - \frac{n_1}{n} h_1 \left(\frac{1}{h_1} - \frac{1}{h_2} \right) \frac{\partial p}{\partial \rho} \right) \right. \\ & \left. + \frac{c_v^{(2)}}{n_2 h_2^2} \left(\frac{\partial p_2}{\partial \rho_2} - \frac{n_2}{n} h_2 \left(\frac{1}{h_2} - \frac{1}{h_1} \right) \frac{\partial p}{\partial \rho} \right) \right]. \end{aligned} \quad (69)$$

This is the general formula for the reactive bulk pressure in a two-component cosmological fluid for small deviations from thermal equilibrium. It is the main result of the paper. By eq.(69) the quantity π_{react} is given in terms of Γ_1 or, via eq.(29), in terms of the production rate Γ . This demonstrates explicitly that particle production in a two-component mixture is equivalent to an effective bulk pressure. The existence of a formula like eq.(69) may be regarded as a semiquantitative justification for using effective viscous pressures in modelling particle creation processes on a phenomenological level. We recall that different from previous studies ([9,10,20–23]) the production process here is nonadiabatic.

In the following we shall apply formula (69) to the out-of-equilibrium decay of a nonrelativistic fluid into a relativistic one.

V. THE OUT-OF-EQUILIBRIUM DECAY OF HEAVY PARTICLES

Decay processes of heavy particles are supposed to play a role, e.g., during baryogenesis where hypothetical heavy bosons decay into quarks and leptons. Another application is the scalar field decay into relativistic particles in the reheating phase of standard inflationary scenarios (see, e.g., [8]).

We assume fluid 1 to be described by the equations of state for nonrelativistic matter, i.e.,

$$\rho_1 = n_1 m + \frac{3}{2} n_1 T_1, \quad p_1 = n_1 T_1, \quad m \gg T, \quad (70)$$

while fluid 2 is a classical relativistic gas:

$$\rho_2 = 3n_2 T_2, \quad p_2 = n_2 T_2. \quad (71)$$

The corresponding enthalpies are

$$h_1 = m + \frac{5}{2} T_1, \quad h_2 = 4T_2, \quad (72)$$

and the specific heats

$$c_v^{(1)} = \frac{3}{2}, \quad c_v^{(2)} = 3. \quad (73)$$

The expressions for $\partial p_A / \partial \rho_A$ and $\partial p / \partial \rho$ become

$$\frac{\partial p_1}{\partial \rho_1} = \frac{2}{3}, \quad \frac{\partial p_2}{\partial \rho_2} = \frac{1}{3}, \quad \frac{\partial p}{\partial \rho} = \frac{2}{3} \frac{n_1 + n_2}{n_1 + 2n_2}. \quad (74)$$

Inserting eqs.(70)-(74) into eq.(69) and taking into account $m \gg T$ we obtain

$$\pi_{react} = \frac{n_1 m}{4} \frac{n_1}{n_1 + 2n_2} \left[1 - \frac{n_1 m}{4n_2 T} \frac{\Gamma_1}{\Theta} \right] \Gamma_1 \eta. \quad (75)$$

Since $\rho + p = \rho_1 + \rho_2 + p_1 + p_2 \approx n_1 m + 4n_2 T$, the ratio $\pi_{react} / (\rho + p)$ is

$$\frac{\pi_{react}}{\rho + p} = \frac{1}{4} \frac{n_1 m}{n_1 m + 4n_2 T} \frac{n_1}{n_1 + 2n_2} \left[1 + \frac{n_1 m}{4n_2 T} \frac{|\Gamma_1|}{\Theta} \right] \Gamma_1 \eta. \quad (76)$$

In order to appropriately apply this formula we have to take into account the limitations following from the linear order restrictions (61) and (62) for the temperature evolution laws. From eq.(13) with (27) we find the linear relations (61) and (62) to make sense for

$$|\Theta - \Gamma_A| (t - t_0) \leq 1. \quad (77)$$

To be cosmologically relevant, we expect $|\Gamma_1|$ to be of the order of Θ , i.e., $-\Gamma_1 = |\Gamma_1| = \alpha\Theta$, where α is a positive constant of the order 1. For component 1 the linear evolution law is valid up to time intervals

$$(t - t_0) \sim \frac{1}{\Theta + |\Gamma_1|} = \frac{\alpha}{\alpha + 1} |\Gamma_1|^{-1}. \quad (78)$$

Assuming furthermore the nonrelativistic component 1 to dominate initially, it follows from eq.(28) that $\Gamma_2 > |\Gamma_1|$, such that $\Gamma_2 > \Theta$ is possible. In the latter case combination of (78) with (77) for $A = 2$ yields

$$\Gamma_2 \sim \frac{\alpha + 2}{\alpha} |\Gamma_1|. \quad (79)$$

Comparing the last expression with eq.(28) we get, by using the equations of state (70) and (71),

$$\rho_1 \sim \frac{4}{3} \frac{\alpha + 2}{\alpha} \rho_2. \quad (80)$$

The limitations of the linear approximation allow the energy density of the matter component to be larger than that of the radiation component by the factor $4(\alpha + 2) / (3\alpha)$. For $\alpha \approx 1$ the ratio $\rho_1 / \rho_2 \approx 4$ is allowed, while for $\alpha \approx 1/4$ we have $\rho_1 / \rho_2 \approx 12$. Since eq.(80) is equivalent to

$$\frac{n_1 m}{4n_2 T} \approx \frac{\alpha + 2}{\alpha}, \quad (81)$$

we find from eq.(75)

$$\pi_{react} = -\frac{n_1 m}{4} \frac{n_1}{n_1 + 2n_2} (\alpha + 3) \alpha \eta \Theta , \quad (82)$$

corresponding to an effective bulk viscosity coefficient ζ_{react}

$$\zeta_{react} = \eta \frac{n_1 m}{4} \frac{n_1}{n_1 + 2n_2} \alpha (\alpha + 3) . \quad (83)$$

It is interesting to compare this bulk viscosity coefficient with the conventional one, characterizing the viscous pressure in a two-fluid system with conserved particle numbers. In general, the latter is given by [7]

$$\zeta = -\tau T \frac{\partial \rho}{\partial T} \left(\frac{\partial p_2}{\partial \rho_2} - \frac{\partial p}{\partial \rho} \right) \left(\frac{\partial p_1}{\partial \rho_1} - \frac{\partial p}{\partial \rho} \right) , \quad (84)$$

which, for matter dominance, coincides in good approximation with Weinberg's expression for the bulk viscosity in radiative hydrodynamics [2,7]. With the equations of state (70) and (71) the expression (84) reduces to

$$\zeta = \frac{\tau}{3} n_2 T \frac{n_1}{n_1 + 2n_2} . \quad (85)$$

Assuming now $\eta \approx \tau$ (see the discussion below eq.(66)) we find

$$\frac{\zeta_{react}}{\zeta} \approx 3 \frac{n_1 m}{4 n_2 T} \alpha (\alpha + 3) , \quad (86)$$

and, with eq.(81),

$$\frac{\zeta_{react}}{\zeta} \approx 3 (\alpha + 2) (\alpha + 3) . \quad (87)$$

For $\alpha \approx 1$ one has $\zeta_{react} \approx 36\zeta$, while for $\alpha \approx 1/4$ the quantity ζ_{react} is more than 20 times larger than ζ . *The reactive bulk viscosity exceeds the conventional one by more than one order of magnitude and, consequently, provides the dominating contribution to the bulk viscous part of the entropy production during the decay process.*

VI. CONCLUSIONS AND OUTLOOK

Particle number nonconserving reactions between two cosmological fluids, each of them perfect on its own, necessarily imply the existence of entropy producing reactive bulk stresses. This result of the present paper provides a semiquantitative justification for the frequently used approach of regarding particle creation processes as phenomenologically equivalent to effective viscous pressures. Different from most of the previous applications, the particle production turned out to be 'nonadiabatic' in the present case, i.e., the entropy per particle of the cosmic fluid as a whole does not remain constant. For small deviations from thermal equilibrium we derived a general formula for the corresponding new kinetic coefficient. The latter was explicitly evaluated for the out-of-equilibrium decay of nonrelativistic fluid particles into relativistic ones. The reactive bulk viscosity coefficient in this case was shown to be larger than the conventional bulk viscosity by more than one order of magnitude.

Our results indicate that bulk pressures are a general cosmological phenomenon. While not taken into account in the standard perfect fluid approaches, one may expect them to have considerably influenced the evolution of the early universe. Of special interest in this respect are far-from-equilibrium situations. The existence of a new type of bulk pressure in addition to the 'conventional' one may shed new light, e.g., on the question whether bulk pressures via their backreaction on the cosmological dynamics might give rise to a phase of inflationary growth of the cosmic scale factor (see [24,11,25] and references therein).

An out-of-equilibrium period that recently has attracted much attention (see, e.g., [26,27]) is the reheating phase of inflationary universe models. On the basis of the temperature evolution law (52) it was shown to be possible to interrelate the description of nonequilibrium processes within the causal, second order Müller-Israel-Stewart theory with particle creation during the 'preheating' stage [28]. A further subject of interest in this connection may be the 'reheating' from primordial black hole evaporation [29].

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